МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ТА ОБЧИСЛЮВАЛЬНІ МЕТОДИ

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OPTIMALITY UNDER NONEMPTY SETS OF LOCAL MIS-EVALUATIONS OF UNCERTAINTIES IN A GENERALIZED MODEL OF FITTING CSS OF SUPPORTS

The Abstract. There is considered a generalized model of fitting cross-section squares (CSS) of supports of the building construction-platform, reflecting the equistrength principle. There has been investigated the case, when there is a nonempty set of indexes of the known values appearing lesser than the corresponding left endpoints, and there is a nonempty set of indexes of the known values appearing greater than the corresponding right endpoints in the given preliminarily evaluated intervals. The investigated case had been interlinked with a local subcase of mis-evaluations, embraced with the proper assertion, which may be evolved further, specifying some wider situations and other cases.

Key words: building construction-platform, fitting cross-section squares (CSS) of supports, interval evaluations, uncertainties, unit-normalization, antagonistic model, over-evaluation, under-evaluation, projector optimal strategy.

Анотація. Розглядається одна узагальнена модель підбору площ поперечного переріза опор будівельної конструкції-платформи, котра відображає принцип рівноміцності. Розглянуто випадок, коли є деяка непорожня множина індексів відомих значень, котрі виявляються меншими за відповідні ліві кінці, і є деяка непорожня множина індексів відомих значень, котрі виявляються більшими за відповідні праві кінці у даних попередньо оцінених інтервалах. Розглянутий випадок пов'язано з локальним підвипадком некоректних оцінок, охопленим відповідним твердженням, котре може бути розвинуте далі, деталізуючи дещо ширші ситуації та інші випадки.

Ключові слова: будівельна конструкція-платформа, підбір площ поперечного переріза опор, інтервальні оцінки, невизначеності, одинична нормалізація, антагоністична модель, переоцінка, недооцінка, оптимальна стратегія проектувальника.

Аннотация. Рассматривается одна обобщённая модель подбора площадей поперечного сечения опор строительной конструкции-платформы, отображающая принцип равнопрочности. Рассмотрен случай, когда существует некоторое непустое множество индексов известных значений, оказывающихся меньшими за соответствующие левые концы, и существует некоторое непустое множество индексов известных значений, оказывающихся бо́льшими за соответствующие правые концы в данных предварительно оцененных интервалах. Рассмотренный случай связано с локальным подслучаем некорректных оценок, охваченным соответствующим утверждением, которое может быть развито далее, детализируя несколько более широкие ситуации и другие случаи.

Ключевые слова: строительная конструкция-платформа, подбор площадей поперечного сечения опор, интервальные оценки, неопределённости, единичная нормализация, антагонистическая модель, переоценка, недооценка, оптимальная стратегия проектировщика.

A problem prevailing review

In getting started, suppose that there is a building construction-platform (BCP), propped up vertically or along horizontally with N supports of some geometry, $N \in \mathbb{N} \setminus \{1\}$, but the total load can be valued only as an interval [1, 2], as well as local loads or pressures, pressing the supports. Under those non-equidistributed potential loads (pressures) on supports there is no possibility to take equiform geometry of supports (unless taking a risk), whereas the main building problem is to ensure safety by minimal resources spendings [1, 3, 4]. The main geometry of supports here is actually their cross-section squares (CSS), acting against the local support loads (LSL), having been pre-evaluated uncertain as intervals. After having unit-normed [5, 6] those interval uncertainties, there is an antagonistic model [5, 7, 8] to ensure additionally minimization of maximal overload [7, 9, 10], in which the second player (SP) is the BCP projector (BCPP), setting up CSS, while different stochastic circumstances, personified by the first player (FP), set on LSL and thus hinder projecting rationally. Although in this model the solution for SP (for BCPP) exists as a pure strategy [7, 8], incorrect pre-evaluation of interval endpoints of those spoken above uncertainties may provoke peculiarities [11, 12] in finding that pure strategy. And narrowly, a one of such peculiarities due to those mis-evaluations is going to get captured.

Available up-to-date origins problem analysis

Recalling then, that the aforementioned model is the convex antagonistic game (AG) with the kernel [7, 13]

$$T(\mathbf{X}, \mathbf{Y}) = T(x_1, x_2, \ldots, x_{N-1}; y_1, y_2, \ldots, y_{N-1}) = \max\left\{ \left\{ \frac{x_j}{y_j^2} \right\}_{j=1}^{N-1}, \frac{1 - \sum_{k=1}^{N-1} x_k}{\left(1 - \sum_{k=1}^{N-1} y_k\right)^2} \right\} \qquad (1)$$

defined on the $\left(2N-2\right)$-dimensional hyperparallelepiped (HP)

$$\boldsymbol{\mathcal{X}} \times \boldsymbol{\mathcal{Y}} = \prod_{p=1}^{2} \left(\prod_{j=1}^{N-1} \left[a_j ; \, b_j \right] \right) \subset \prod_{d=1}^{2N-2} \left(0; \, 1\right) \subset \prod_{d=1}^{2N-2} \left[0; \, 1\right] \subset \mathbb{R}^{2N-2} \tag{2}$$

as the Cartesian product of HP

$$\boldsymbol{\mathcal{X}} = \prod_{j=1}^{N-1} \left[a_j ; \, b_j \right] \subset \prod_{j=1}^{N-1} \left(0; \, 1\right) \subset \prod_{j=1}^{N-1} \left[0; \, 1\right] \subset \mathbb{R}^{N-1} \tag{3}$$

of the FP pure strategies (normed LSL) and of HP

$$\boldsymbol{\mathcal{Y}} = \prod_{j=1}^{N-1} \left[a_j ; \, b_j \right] \subset \prod_{j=1}^{N-1} \left(0; \, 1\right) \subset \prod_{j=1}^{N-1} \left[0; \, 1\right] \subset \mathbb{R}^{N-1} \tag{4}$$

of the SP pure strategies (normed CSS), where the number of supports $N \in \mathbb{N} \setminus \left\{1\right\}$,

$$\mathbf{X} = \left[x_1 \quad x_2 \quad \cdots \quad x_{N-2} \quad x_{N-1} \right] \in \boldsymbol{\mathcal{X}} , \quad \mathbf{Y} = \left[y_1 \quad y_2 \quad \cdots \quad y_{N-2} \quad y_{N-1} \right] \in \boldsymbol{\mathcal{Y}} ,$$

and the variable

$$x_j \in \left[a_j ; \, b_j \right] \subset \left(0; \, 1\right) \text{ by } a_j < b_j \quad \forall \; j = \overline{1, \, N-1} \tag{5}$$

is the normed j-th support load, the variable

$$y_j \in \left[a_j ; \, b_j \right] \subset \left(0; \, 1\right) \text{ by } a_j < b_j \quad \forall \; j = \overline{1, \, N-1} \tag{6}$$

is the normed j-th support cross-section square, and totals

$$\sum_{k=1}^{N} x_k = 1 , \quad \sum_{k=1}^{N} y_k = 1 \tag{7}$$

due to the unit-normalization over LSL and CSS. In this AG (1) — (7) BCPP has the optimal pure strategy (OPS)

$$\mathbf{Y}_{*} = \left[y_1^{*} \quad y_2^{*} \quad \cdots \quad y_{N-2}^{*} \quad y_{N-1}^{*} \right] \in \boldsymbol{\mathcal{Y}} \tag{8}$$

as $\left(N-1\right)$-dimensional point of HP (4), that is the condition

$$y_j^{*} \in \left[a_j ; \, b_j \right] \quad \forall \; j = \overline{1, \, N-1} \tag{9}$$

stands clear. Here the $\left(N-1\right)$-dimensional point (8) with components (9)

$$\mathbf{Y}_* = \left[y_1^* \quad y_2^* \quad \cdots \quad y_{N-2}^* \quad y_{N-1}^* \right] \in \arg \min_{\mathbf{Y} \in \prod_{i=1}^{N-1} [a_i\,;\, b_i]} \left\{ \max_{\mathbf{X} \in \prod_{j=1}^{N-1} [a_j\,;\, b_j]} T(\mathbf{X}, \mathbf{Y}) \right\} =$$

$$= \arg \min_{\mathbf{Y} \in \prod_{i=1}^{N-1} [a_i\,;\, b_i]} \left\{ \max_{\mathbf{X} \in \prod_{j=1}^{N-1} [a_j\,;\, b_j]} \left(\max \left\{ \left\{ \frac{x_j}{y_j^2} \right\}_{j=1}^{N-1}, \frac{1 - \sum_{k=1}^{N-1} x_k}{\left(1 - \sum_{k=1}^{N-1} y_k\right)^2} \right\} \right) \right\} =$$

$$= \arg \min_{\mathbf{Y} \in \prod_{i=1}^{N-1} [a_i\,;\, b_i]} \left\{ \max \left\{ \left\{ \frac{b_j}{y_j^2} \right\}_{j=1}^{N-1}, \frac{1 - \sum_{k=1}^{N-1} a_k}{\left(1 - \sum_{k=1}^{N-1} y_k\right)^2} \right\} \right\} \tag{10}$$

and the regular equality

$$\frac{b_j}{y_j^2} = \frac{1 - \sum_{k=1}^{N-1} a_k}{\left(1 - \sum_{k=1}^{N-1} y_k\right)^2} \quad \forall\ j = \overline{1,\, N-1} \tag{11}$$

gives directly values

$$y_j = \frac{\sqrt{b_j}}{\sum_{k=1}^{N-1} \sqrt{b_k} + \sqrt{1 - \sum_{k=1}^{N-1} a_k}} \quad \forall\ j = \overline{1,\, N-1} \tag{12}$$

as the roots of (11), which by

$$\frac{\sqrt{b_j}}{\sum_{k=1}^{N-1} \sqrt{b_k} + \sqrt{1 - \sum_{k=1}^{N-1} a_k}} \in \left[a_j;\, b_j \right] \quad \forall\ j = \overline{1,\, N-1} \tag{13}$$

are components $\left\{ y_j^* \right\}_{j=1}^{N-1}$ of OPS (8). But if the membership (13) fails, that is there is at least $\exists\ q \in \left\{ \overline{1,\, N-1} \right\}$ such that

$$\frac{\sqrt{b_q}}{\sum_{k=1}^{N-1} \sqrt{b_k} + \sqrt{1 - \sum_{k=1}^{N-1} a_k}} \notin \left[a_q;\, b_q \right] \quad \text{for} \quad \exists\ q \in \left\{ \overline{1,\, N-1} \right\}, \tag{14}$$

then the equality (11) cannot be fulfilled within HP (4) and the directly obtained values (12), speaking strictly, aren’t components $\left\{y_j^*\right\}_{j=1}^{N-1}$ of OPS (8). Such cases had been investigated through papers [11, 12, 14, 15], but the question on OPS (8) for BCPP by $N \in \mathbb{N} \setminus \{1\}$ generally stays open. Moreover, there are two types of incorrectness in pre-evaluating endpoints $\left\{a_j\right\}_{j=1}^{N-1}$ and $\left\{b_j\right\}_{j=1}^{N-1}$ (two types of mis-evaluations), driving to failure in the membership (13) with

$$\frac{\sqrt{b_r}}{\sum_{k=1}^{N-1}\sqrt{b_k}+\sqrt{1-\sum_{k=1}^{N-1}a_k}} < a_r \quad \text{for} \quad r \in A \subset \left\{\overline{1,\, N-1}\right\} \tag{15}$$

and

$$\frac{\sqrt{b_t}}{\sum_{k=1}^{N-1}\sqrt{b_k}+\sqrt{1-\sum_{k=1}^{N-1}a_k}} > b_t \quad \text{for} \quad t \in B \subset \left\{\overline{1,\, N-1}\right\} \tag{16}$$

at the nonempty subsets A and B what is going to become the investigation object [15], whereas investigation of just (15) or (16) separately naturally comes easier or even trivial [14, 16]. However, this heterogeneity of mis-evaluations supposes pretty long AG (1) — (7) solution statement [12, 15], so there will be developed only a local identity subcase.

Object and goal

AG (1) — (7) conditions are supplemented with that $A \neq \varnothing$ and $B \neq \varnothing$, where

$$\frac{\sqrt{b_i}}{\sum_{k=1}^{N-1}\sqrt{b_k}+\sqrt{1-\sum_{k=1}^{N-1}a_k}} \in \left[a_i;\, b_i\right] \quad \forall\; i \in \left\{\overline{1,\, N-1}\right\} \setminus \left\{A \bigcup B\right\}. \tag{17}$$

After having got started with those objectified conditions of over-evaluations (15) and under-evaluations (16) of uncertainties (mis-evaluations) $\left\{\left[a_j;\, b_j\right]\right\}_{j=1}^{N-1}$ with (17), there stands the goal to find components $\left\{y_j^*\right\}_{j=1}^{N-1}$ of OPS (8) for BCPP within a local identity subcase. This case corresponding assertion is to be emphasized as the theorem for that generalized antagonistic model. It will allow to have CSS of BCP fitted rationally, what prevents the BCP maximal overload and improves exploiting it under mis-evaluations of uncertainties $\left\{\left[a_j;\, b_j\right]\right\}_{j=1}^{N-1}$.

OPS (8) for BCPP in the game (1) — (7) under (15) — (17) with sets $A \neq \varnothing$ and $B \neq \varnothing$ for a local subcase

Theorem. In AG with the kernel (1) on HP (2) at (3) — (7) for $N \in \mathbb{N} \setminus \{1, 2\}$ under conditions (15) — (17) by

$$\max_{g\in B}\left\{\frac{1}{b_g}\right\}=\frac{1-\sum_{k=1}^{N-1}a_k}{\left(1-\sum_{r\in A\subset\left\{\overline{1,\, N-1}\right\}}a_r-\sum_{t\in B\subset\left\{\overline{1,\, N-1}\right\}}b_t-\left(\sum_{j=1}^{N-1}\sqrt{b_j}+\sqrt{1-\sum_{j=1}^{N-1}a_j}\right)^{-1}\sum_{k\in\left\{\overline{1,\, N-1}\right\}\backslash\{A\cup B\}}\sqrt{b_k}\right)^2} \tag{18}$$

SP has OPS (8) with the $t_{\max}$ -th component

$$y^*_{t_{\max}}=b_{t_{\max}}\quad\forall\ t_{\max}\in B_{\max}\subset B\subset\left\{\overline{1,\, N-1}\right\}\quad\text{for }\frac{1}{b_{t_{\max}}}=\max_{g\in B}\left\{\frac{1}{b_g}\right\},\quad B_{\max}=\arg\max_{g\in B}\left\{\frac{1}{b_g}\right\}, \tag{19}$$

and if

$$\sum_{k=1}^{N-1}b_k\leqslant 1-\sqrt{b_{t_{\max}}\left(1-\sum_{k=1}^{N-1}a_k\right)} \tag{20}$$

then the j -th component

$$y^*_j\in\left[\frac{1}{2}\left(\sqrt{b_{t_{\max}}b_j}+a_j+\left(\sqrt{b_{t_{\max}}b_j}-a_j\right)\operatorname{sign}\left(\sqrt{b_{t_{\max}}b_j}-a_j\right)\right);\ b_j\right]\quad\forall\ j\in\left\{\overline{1,\, N-1}\right\}\backslash B_{\max}, \tag{21}$$

but if

$$\sum_{k=1}^{N-1}b_k>1-\sqrt{b_{t_{\max}}\left(1-\sum_{k=1}^{N-1}a_k\right)} \tag{22}$$

then the j -th component

$$y^*_j\in\left[\frac{1}{2}\left(\sqrt{b_{t_{\max}}b_j}+a_j+\left(\sqrt{b_{t_{\max}}b_j}-a_j\right)\operatorname{sign}\left(\sqrt{b_{t_{\max}}b_j}-a_j\right)\right);\ y_j^{\langle\max\rangle}\right]\quad\forall\ j\in\left\{\overline{1,\, N-1}\right\}\backslash B_{\max} \tag{23}$$

for

$$y_j^{\langle\max\rangle}\in\left[\frac{1}{2}\left(\sqrt{b_{t_{\max}}b_j}+a_j+\left(\sqrt{b_{t_{\max}}b_j}-a_j\right)\operatorname{sign}\left(\sqrt{b_{t_{\max}}b_j}-a_j\right)\right);\ b_j\right]\quad\forall\ j\in\left\{\overline{1,\, N-1}\right\}\backslash B_{\max} \tag{24}$$

at

$$\sum_{k\in\left\{\overline{1,\, N-1}\right\}\backslash B_{\max}}y_k^{\langle\max\rangle}=1-\sum_{t\in B_{\max}}b_t-\sqrt{b_{t_{\max}}\left(1-\sum_{k=1}^{N-1}a_k\right)}. \tag{25}$$

Proof. Up with the stated (18), taking

$$y_r = a_r > \frac{\sqrt{b_r}}{\sum_{k=1}^{N-1}\sqrt{b_k} + \sqrt{1-\sum_{k=1}^{N-1} a_k}} \quad \forall\ r \in A \tag{26}$$

and

$$y_t = b_t < \frac{\sqrt{b_t}}{\sum_{k=1}^{N-1}\sqrt{b_k} + \sqrt{1-\sum_{k=1}^{N-1} a_k}} \quad \forall\ t \in B \tag{27}$$

for comparing parts $\left\{\frac{b_j}{y_j^2}\right\}_{j=1}^{N-1}$ of the equality (11), get

$$\max_{g\in B}\left\{\frac{1}{b_g}\right\} = \frac{1-\sum_{k=1}^{N-1} a_k}{\left(1-\sum_{r\in A\subset\left\{\overline{1,\,N-1}\right\}} a_r - \sum_{t\in B\subset\left\{\overline{1,\,N-1}\right\}} b_t - \left(\sum_{j=1}^{N-1}\sqrt{b_j}+\sqrt{1-\sum_{j=1}^{N-1}a_j}\right)^{-1}\sum_{k\in\left\{\overline{1,\,N-1}\right\}\setminus\{A\cup B\}}\sqrt{b_k}\right)^2} >$$

$$> \frac{1}{b_t} > \frac{b_i}{\left(\sqrt{b_i}\left(\sum_{k=1}^{N-1}\sqrt{b_k}+\sqrt{1-\sum_{k=1}^{N-1}a_k}\right)^{-1}\right)^2} = \left(\sum_{k=1}^{N-1}\sqrt{b_k}+\sqrt{1-\sum_{k=1}^{N-1}a_k}\right)^2 > \frac{b_r}{a_r^2}$$

$$\forall\ t \in B\setminus B_{\max},\quad \forall\ i \in \left\{\overline{1,\,N-1}\right\}\setminus\{A\cup B\},\quad \forall\ r \in A. \tag{28}$$

For finding OPS (8) components $\left\{y_i^*\right\}_{i\in\left\{\overline{1,\,N-1}\right\}\setminus\{A\cup B\}}$, $\left\{y_r^*\right\}_{r\in A}$ and $\left\{y_t^*\right\}_{t\in B}$, it is obvious from (28) that the optimal game value $v_* = \frac{1}{b_{t_{\max}}}$, being reached in particular on the $t_{\max}$-th component (19). If it were $y_{t_{\max}}^* < b_{t_{\max}}$ then the game value would have increased what violates the SP optimality principle, so (19) is uniquely true. Due to (18) and (28) SP may use such components $\left\{y_j^*\right\}_{j\in\left\{\overline{1,\,N-1}\right\}\setminus B_{\max}}$ that there would be non-strict inequalities

$$\frac{1}{b_{t_{\max}}} \geqslant \frac{1-\sum_{k=1}^{N-1} a_k}{\left(1-\sum_{t\in B_{\max}} b_t - \sum_{k\in\left\{\overline{1,\,N-1}\right\}\setminus B_{\max}} y_k^*\right)^2} \tag{29}$$

and

$$\frac{1}{b_{t_{\max}}} \geqslant \frac{b_j}{\left(y_j^*\right)^2} \quad \forall\ j \in \left\{\overline{1,\ N-1}\right\} \setminus B_{\max} \tag{30}$$

simultaneously. Then from (29) get the nonstrict inequality

$$1 - \sum_{t \in B_{\max}} b_t - \sum_{k \in \left\{\overline{1,\ N-1}\right\} \setminus B_{\max}} y_k^* \geqslant \sqrt{b_{t_{\max}} \left(1 - \sum_{k=1}^{N-1} a_k\right)},$$

$$\sum_{k \in \left\{\overline{1,\ N-1}\right\} \setminus B_{\max}} y_k^* \leqslant 1 - \sum_{t \in B_{\max}} b_t - \sqrt{b_{t_{\max}} \left(1 - \sum_{k=1}^{N-1} a_k\right)} \tag{31}$$

and from (30) get the nonstrict inequality

$$y_j^* \geqslant \sqrt{b_{t_{\max}} b_j} \quad \forall\ j \in \left\{\overline{1,\ N-1}\right\} \setminus B_{\max}, \tag{32}$$

where it is obligatory to control subcases with $\sqrt{b_{t_{\max}} b_j} \geqslant a_j$ and $\sqrt{b_{t_{\max}} b_j} < a_j$, which may occur both enough. By (20) the condition (31) or the initial condition (29) is ever true for any components $\left\{y_j^*\right\}_{j \in \left\{\overline{1,\ N-1}\right\} \setminus B_{\max}}$ satisfying (32) with controlling whether $\sqrt{b_{t_{\max}} b_j} \geqslant a_j$ or $\sqrt{b_{t_{\max}} b_j} < a_j$, what is stated compactly as (21). Inversely, by (22) each j-th component y_j^* in (23) under condition (32) is upper-limited with such $y_j^{\langle \max \rangle}$ in (24) that (31) turns into equality, what corresponds to the statement (25), being the boundary sum. The theorem has been proved.

The usage of the proved theorem lies in calculating the OPS (8) continuum, generated with components $\left\{y_j^*\right\}_{j \in \left\{\overline{1,\ N-1}\right\} \setminus B_{\max}}$ as (21) or (23) for (24) at (25), where BCPP may select freely just as needed. For instance, if

$$\left\{\left[a_j;\ b_j\right]\right\}_{j=1}^{3} = \left\{[0.35;\ 0.36],\ [0.02;\ 0.15],\ [0.03;\ 0.2]\right\} \tag{33}$$

for a classic BCP, propped up vertically or along horizontally with four supports of some geometry, then the values (12)

$$\frac{\sqrt{b_1}}{\sum_{k=1}^{3} \sqrt{b_k} + \sqrt{1 - \sum_{k=1}^{3} a_k}} = \frac{\sqrt{0.36}}{\sqrt{0.36} + \sqrt{0.15} + \sqrt{0.2} + \sqrt{0.6}} < 0.271603 < 0.35 = a_1, \tag{34}$$

$$\frac{\sqrt{b_2}}{\sum_{k=1}^{3} \sqrt{b_k} + \sqrt{1 - \sum_{k=1}^{3} a_k}} = \frac{\sqrt{0.15}}{\sqrt{0.36} + \sqrt{0.15} + \sqrt{0.2} + \sqrt{0.6}} > 0.1753188 > 0.15 = b_2, \tag{35}$$

$$\frac{\sqrt{b_3}}{\sum_{k=1}^{3}\sqrt{b_k}+\sqrt{1-\sum_{k=1}^{3}a_k}}=\frac{\sqrt{0.2}}{\sqrt{0.36}+\sqrt{0.15}+\sqrt{0.2}+\sqrt{0.6}}>0.20244>0.2=b_3 . \quad (36)$$

So, under uncertainties (33), given the values (34) — (36), here have (15) and (16) with sets $A=\{1\}$ and $B=\{2, 3\}$ for the local subcase (18): truly,

$$\max_{g\in B=\{2,\,3\}}\left\{\frac{1}{b_g}\right\}=\max\left\{\frac{1}{b_2},\,\frac{1}{b_3}\right\}=\frac{1}{b_2}=\frac{20}{3}=$$

$$=\frac{1-\sum_{k=1}^{3}a_k}{\left(1-\sum_{r\in A=\{1\}}a_r-\sum_{t\in B=\{2,\,3\}}b_t-\left(\sum_{j=1}^{3}\sqrt{b_j}+\sqrt{1-\sum_{j=1}^{3}a_j}\right)^{-1}\sum_{k\in\{\overline{1,\,3}\}\setminus\{A\cup B\}=\varnothing}\sqrt{b_k}\right)^2}=$$

$$=\frac{1-\sum_{k=1}^{3}a_k}{\left(1-a_1-b_2-b_3\right)^2}=\frac{0.6}{\left(1-0.7\right)^2}=\frac{0.6}{0.09}=\frac{20}{3} . \quad (37)$$

As for uncertainties (33)

$$\sum_{k=1}^{3}b_k=0.71>1-\sqrt{b_{t_{\max}}\left(1-\sum_{k=1}^{3}a_k\right)}=$$

$$=1-\sqrt{b_2\left(1-\sum_{k=1}^{3}a_k\right)}=1-\sqrt{0.15\left(1-0.4\right)}=1-0.3=0.7 \quad (38)$$

then due to (19) and (22) the BCPP solution is

$$y_2^*=b_2,\ \ y_1^*\in\left[0.35;\ y_1^{\langle\max\rangle}\right],\ \ y_3^*\in\left[\sqrt{0.03};\ y_3^{\langle\max\rangle}\right]\ \text{ for }\ y_1^{\langle\max\rangle}\in\left[0.35;\ 0.36\right]$$

$$\text{and }\ y_3^{\langle\max\rangle}\in\left[\sqrt{0.03};\ 0.2\right]\ \text{ at}$$

$$y_1^{\langle\max\rangle}+y_3^{\langle\max\rangle}=1-b_2-\sqrt{b_2\left(1-\sum_{k=1}^{3}a_k\right)}=1-0.15-\sqrt{0.15\left(1-0.4\right)}=0.85-0.3=0.55 , \quad (39)$$

though components $\left\{y_j^*\right\}_{j=1}^{3}$ in (39) constitute actually a continuum of OPS (8).

Concluding the investigation and outlining the further work

Undoubtedly, the considered locally antagonistic model as AG (1) — (7) can't rival models of actually creating, designing real building construction. Nevertheless, fitting CSS of supports of BCP is a very important task for saving building resources, minimizing geometrical dimensions and increasing capacity [2, 4, 6, 17 — 21]. The task of that fitting has been made more precise here, within the given paper, with having found OPS (8) for BCPP under mis-evaluations (15) — (17), occurring pretty frequently. Commonly, the stated paper results can be applied to other economic-ecologic-social and technical problems, where the equistrength principle is reflected as the ratio of some action (load) against the squared counteraction [1, 2, 7, 21, 22]. In perspective, there in AG (1) — (7) under mis-evaluations (15) — (17) should be investigated other wider situations and cases, generating, furthermore, continuums of OPS (8). Single-element-selection from these continuums, one of which just has been displayed in (39) according to (37) and (38), may put a furthered problem, if BCPP is not able to accomplish such selection heuristically.

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